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Sum Rules of Neutrino Masses and CP Violation in the Four-Neutrino Mixing Scheme

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Abstract

We show that the commutator of lepton mass matrices is invariant under terrestial matter effects in the four-neutrino mixing scheme. A set of model-independent sum rules for neutrino masses, which may be generalized to hold for an arbitrary number of neutrino families, are for the first time uncovered. Useful sum rules for the rephasing-invariant measures of leptonic CP violation have also been found. Finally we present a generic formula of T-violating asymmetries and expect it to be applicable to the future long-baseline neutrino oscillation experiments.

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1 Recently the Super-Kamiokande Collaboration has reported some robust evidence for atmospheric and solar neutrino oscillations [1]. In addition, $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ oscillations have been observed by the LSND Collaboration [2]. A simultaneous interpretation of solar, atmospheric and LSND neutrino oscillation data has to invoke the existence of a light sterile neutrino [3], because they involve three distinct mass-squared differences ($\Delta m^2_{\rm sun} \ll \Delta m^2_{\rm atm} \ll \Delta m^2_{\rm LSND}$). In the four-neutrino mixing scheme, CP violation is generally expected to manifest itself. To measure leptonic CP- and T-violating effects needs a new generation of accelerator neutrino experiments with very long baselines, including the proposed neutrino factories [4]. In such long-baseline experiments the earth-induced matter effects, which are likely to deform the neutrino oscillation patterns in vacuum and to fake the genuine CP- and T-violating signals, must be taken into account.

To pin down the underlying dynamics of lepton mass generation and CP violation relies crucially upon how accurately the fundamental parameters of lepton flavor mixing can be measured and disentangled from terrestrial matter effects [5]. It is therefore desirable to explore possible model-independent relations between the effective neutrino masses in matter and the genuine neutrino masses in vacuum. It is also useful to establish some model-independent relations between the rephasing-invariant measures of CP violation in matter and those in vacuum. So far, however, the work of this nature has been lacking.

This paper aims to show that the commutator of lepton mass matrices in the four-neutrino mixing scheme is invariant under terrestrial matter effects. As a consequence, some concise sum rules of neutrino masses can be obtained model-independently. Such sum rules can even be generalized to hold for an arbitrary number of neutrino families. Another set of sum rules are derived as well for the rephasing-invariant measures of leptonic CP violation in matter. Finally we present a generic formula of T-violating asymmetries, which is applicable in particular to the future long-baseline neutrino oscillation experiments.

2 The phenomenon of lepton flavor mixing arises from the mismatch between the diagonalization of the charged lepton mass matrix M_l and that of the neutrino mass matrix M_ν in an arbitrary flavor basis. Without loss of generality, one may choose to identify the flavor eigenstates of charged leptons with their mass eigenstates. In this specific basis, where M_l is diagonal, the lepton flavor mixing matrix V links the neutrino flavor eigenstates directly to the neutrino mass eigenstates. For the admixture of one sterile (ν_s) and three active (ν_e, ν_μ, ν_τ) neutrinos, the explicit form of V reads

$$\begin{pmatrix} \nu_s \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{s0} & V_{s1} & V_{s2} & V_{s3} \\ V_{e0} & V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 0} & V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 0} & V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_0 \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} , \tag{1}$$

where ν_i (for $i = 0, 1, 2, 3$) denote the mass eigenstates of four neutrinos. The effective Hamiltonian responsible for the propagation of neutrinos in vacuum can be written as [6]

$$\mathcal{H}_{\rm eff} = \frac{1}{2E} \left(M_\nu M^\dagger_\nu \right) = \frac{1}{2E} \left(V D^2_\nu V^\dagger \right) , \tag{2}$$

where $D_\nu \equiv {\rm Diag}\{m_0, m_1, m_2, m_3\}$, m_i are the neutrino mass eigenvalues, and $E \gg m_i$ denotes the neutrino beam energy. Note that it is $M_\nu M^\dagger_\nu$ (instead of $M^\dagger_\nu M_\nu$) that associates with $\mathcal{H}_{\rm eff}$ [7], no matter whether neutrinos are Dirac or Majorana particles. When

active neutrinos travel through a normal material medium (e.g., the earth), which consists of electrons but of no muons or taus, they encounter both charged- and neutral-current interactions with electrons. The neutral-current interaction is universal for ν_e, ν_μ and ν_τ, while the charged-current interaction is associated only with ν_e. Their effects on the mixing and propagating features of neutrinos have to be taken into account in all long-baseline neutrino oscillation experiments. Let us use $\tilde{M}_\nu$ and $\tilde{V}$ to denote the effective neutrino mass matrix and the effective flavor mixing matrix in matter, respectively. Then the effective Hamiltonian responsible for the propagation of neutrinos in matter can be written as

$$\tilde{\mathcal{H}}_{\rm eff} \;=\; \frac{1}{2E}\left(\tilde{M}_\nu \tilde{M}_\nu^\dagger\right) \;=\; \frac{1}{2E}\left(\tilde{V}\tilde{D}_\nu^2\tilde{V}^\dagger\right) \;, \tag{3}$$

where $\tilde{D}_\nu \equiv {\rm Diag}\{\tilde{m}_0, \tilde{m}_1, \tilde{m}_2, \tilde{m}_3\}$, and $\tilde{m}_i$ are the effective neutrino mass eigenvalues in matter. The deviation of $\tilde{\mathcal{H}}_{\rm eff}$ from $\mathcal{H}_{\rm eff}$ is given by

$$\Delta\mathcal{H}_{\rm eff} \;\equiv\; \tilde{\mathcal{H}}_{\rm eff} - \mathcal{H}_{\rm eff} = \begin{pmatrix} a' & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \;, \tag{4}$$

where $a = \sqrt{2}\, G_{\rm F} N_e$ and $a' = \sqrt{2}\, G_{\rm F} N_n/2$ with N_e and N_n being the background densities of electrons and neutrons [8], respectively. In the following we assume a constant earth density profile (i.e., $N_e =$ constant and $N_n =$ constant), which is a very good approximation for all of the presently-proposed long-baseline neutrino experiments.

Now let us introduce the commutators of 4×4 lepton mass matrices to describe the flavor mixing of one sterile and three active neutrinos. Without loss of any generality, we continue to work in the afore-chosen flavor basis, where M_l takes the diagonal form $D_l = {\rm Diag}\{m_s, m_e, m_\mu, m_\tau\}$ with $m_s = 0$. Note that we have assumed the $(1,1)$ element of D_l to be zero, because there is no counterpart of the sterile neutrino ν_s in the charged lepton sector. We shall see later on that our physical results are completely independent of m_s, no matter what value it may take. The commutator of lepton mass matrices in vacuum and that in matter can then be defined as

$$\begin{aligned} C &\equiv i\left[M_\nu M_\nu^\dagger \;, M_l M_l^\dagger\right] = i\left[V D_\nu^2 V^\dagger, D_l^2\right] \;, \\ \tilde{C} &\equiv i\left[\tilde{M}_\nu \tilde{M}_\nu^\dagger \;, M_l M_l^\dagger\right] = i\left[\tilde{V}\tilde{D}_\nu^2\tilde{V}^\dagger, D_l^2\right] \;. \end{aligned} \tag{5}$$

Obviously C and $\tilde{C}$ are traceless Hermitian matrices. In terms of neutrino masses and flavor mixing matrix elements, we obtain the explicit expressions of C and $\tilde{C}$ as follows:

$$\begin{aligned} C &= i\begin{pmatrix} 0 & \Delta_{es}Z_{se} & \Delta_{\mu s}Z_{s\mu} & \Delta_{\tau s}Z_{s\tau} \\ \Delta_{se}Z_{es} & 0 & \Delta_{\mu e}Z_{e\mu} & \Delta_{\tau e}Z_{e\tau} \\ \Delta_{s\mu}Z_{\mu s} & \Delta_{e\mu}Z_{\mu e} & 0 & \Delta_{\tau\mu}Z_{\mu\tau} \\ \Delta_{s\tau}Z_{\tau s} & \Delta_{e\tau}Z_{\tau e} & \Delta_{\mu\tau}Z_{\tau\mu} & 0 \end{pmatrix} \;, \\ \tilde{C} &= i\begin{pmatrix} 0 & \Delta_{es}\tilde{Z}_{se} & \Delta_{\mu s}\tilde{Z}_{s\mu} & \Delta_{\tau s}\tilde{Z}_{s\tau} \\ \Delta_{se}\tilde{Z}_{es} & 0 & \Delta_{\mu e}\tilde{Z}_{e\mu} & \Delta_{\tau e}\tilde{Z}_{e\tau} \\ \Delta_{s\mu}\tilde{Z}_{\mu s} & \Delta_{e\mu}\tilde{Z}_{\mu e} & 0 & \Delta_{\tau\mu}\tilde{Z}_{\mu\tau} \\ \Delta_{s\tau}\tilde{Z}_{\tau s} & \Delta_{e\tau}\tilde{Z}_{\tau e} & \Delta_{\mu\tau}\tilde{Z}_{\tau\mu} & 0 \end{pmatrix} \;, \end{aligned} \tag{6}$$

where $\Delta_{\alpha\beta} \equiv m_\alpha^2 - m_\beta^2$ for $\alpha \neq \beta$ running over (s, e, μ, τ), and

$$
\begin{aligned}
Z_{\alpha\beta} &\equiv \sum_{i=0}^{3} \left(m_i^2 V_{\alpha i} V_{\beta i}^* \right) \; , \\
\tilde{Z}_{\alpha\beta} &\equiv \sum_{i=0}^{3} \left(\tilde{m}_i^2 \tilde{V}_{\alpha i} \tilde{V}_{\beta i}^* \right) \; .
\end{aligned}
\tag{7}
$$

One can see that $\Delta_{\beta\alpha} = -\Delta_{\alpha\beta}$, $Z_{\beta\alpha} = Z_{\alpha\beta}^*$ and $\tilde{Z}_{\beta\alpha} = \tilde{Z}_{\alpha\beta}^*$ hold.

To find out how $\tilde{Z}_{\alpha\beta}$ is connected with $Z_{\alpha\beta}$, we need to establish the relation between $\tilde{C}$ and C. Taking Eqs. (2), (3) and (4) into account, we immediately obtain

$$
\tilde{C} \; = \; 2iE \left[\tilde{\mathcal{H}}_{\rm eff} \; , D_l^2 \right] = C + 2iE \left[\Delta \mathcal{H}_{\rm eff} \; , D_l^2 \right] = C \; . \tag{8}
$$

This interesting result indicates that *the commutator of lepton mass matrices in vacuum is invariant under terrestrial matter effects*. As a straightforward consequence of $\tilde{C} = C$, we arrive at $\tilde{Z}_{\alpha\beta} = Z_{\alpha\beta}$ from Eq. (6). Then a set of concise sum rules of neutrino masses emerge:

$$
\sum_{i=0}^{3} \left(\tilde{m}_i^2 \tilde{V}_{\alpha i} \tilde{V}_{\beta i}^* \right) \; = \; \sum_{i=0}^{3} \left(m_i^2 V_{\alpha i} V_{\beta i}^* \right) \; ; \tag{9}
$$

or equivalently

$$
\sum_{i=1}^{3} \left(\tilde{\Delta}_{i0} \tilde{V}_{\alpha i} \tilde{V}_{\beta i}^* \right) \; = \; \sum_{i=1}^{3} \left(\Delta_{i0} V_{\alpha i} V_{\beta i}^* \right) \; , \tag{10}
$$

where $\Delta_{i0} \equiv m_i^2 - m_0^2$ and $\tilde{\Delta}_{i0} \equiv \tilde{m}_i^2 - \tilde{m}_0^2$ for $i = 1, 2, 3$. It becomes obvious that the validity of Eq. (9) or Eq. (10) has nothing to do with the assumption of $m_s = 0$ in the charged lepton sector. Although we have derived these sum rules in the four-neutrino mixing scheme, they may simply be generalized to hold for an arbitrary number of neutrino families.

It should be noted that $Z_{\alpha\beta}$ and $\tilde{Z}_{\alpha\beta}$ are sensitive to a redefinition of the phases of charged lepton fields. The simplest rephasing-invariant equality is of course $|\tilde{Z}_{\alpha\beta}| = |Z_{\alpha\beta}|$. For the description of CP or T violation in neutrino oscillations, we are more interested in the following rephasing-invariant relationship:

$$
\tilde{Z}_{\alpha\beta} \tilde{Z}_{\beta\gamma} \tilde{Z}_{\gamma\alpha} \; = \; Z_{\alpha\beta} Z_{\beta\gamma} Z_{\gamma\alpha} \; , \tag{11}
$$

for $\alpha \neq \beta \neq \gamma$ running over (s, e, μ, τ). As one can see later on, the imaginary parts of $Z_{\alpha\beta} Z_{\beta\gamma} Z_{\gamma\alpha}$ and $\tilde{Z}_{\alpha\beta} \tilde{Z}_{\beta\gamma} \tilde{Z}_{\gamma\alpha}$ are related respectively to leptonic CP violation in vacuum and that in matter.

It should also be noted that the results obtained above are only valid for neutrinos propagating in vacuum and in matter. As for antineutrinos, the corresponding formulas can straightforwardly be written out from Eqs. (3) – (11) through the replacements $V \Longrightarrow V^*$, $a \Longrightarrow -a$ and $a' \Longrightarrow -a'$.

3 In terms of the matrix elements of V or $\tilde{V}$, one may define the rephasing-invariant measures of CP violation as follows [9]:

$$
\begin{aligned}
J^{ij}_{\alpha\beta} &\equiv {\rm Im}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right) \; , \\
\tilde{J}^{ij}_{\alpha\beta} &\equiv {\rm Im}\left(\tilde{V}_{\alpha i}\tilde{V}_{\beta j}\tilde{V}^*_{\alpha j}\tilde{V}^*_{\beta i}\right) \; ,
\end{aligned}
\tag{12}
$$

where the Greek subscripts ($\alpha \neq \beta$) run over (s, e, μ, τ), and the Latin superscripts ($i \neq j$) run over $(0, 1, 2, 3)$. Of course, $J^{ii}_{\alpha\beta} = J^{ij}_{\alpha\alpha} = 0$ and $\tilde{J}^{ii}_{\alpha\beta} = \tilde{J}^{ij}_{\alpha\alpha} = 0$ hold by definition. With the help of the unitarity of V or $\tilde{V}$, one may straightforwardly obtain the following correlation equations of $J^{ij}_{\alpha\beta}$ and $\tilde{J}^{ij}_{\alpha\beta}$:

$$
\begin{aligned}
\sum_i J^{ij}_{\alpha\beta} = \sum_j J^{ij}_{\alpha\beta} &= 0 \; , \\
\sum_i \tilde{J}^{ij}_{\alpha\beta} = \sum_j \tilde{J}^{ij}_{\alpha\beta} &= 0 \; ;
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
\sum_\alpha J^{ij}_{\alpha\beta} = \sum_\beta J^{ij}_{\alpha\beta} &= 0 \; , \\
\sum_\alpha \tilde{J}^{ij}_{\alpha\beta} = \sum_\beta \tilde{J}^{ij}_{\alpha\beta} &= 0 \; .
\end{aligned}
\tag{14}
$$

Note that there are totally nine independent $J^{ij}_{\alpha\beta}$ (or $\tilde{J}^{ij}_{\alpha\beta}$) in the four-neutrino mixing scheme under discussion. If only the flavor mixing of three active neutrinos is taken into account, there will be a single independent $J^{ij}_{\alpha\beta}$ (or $\tilde{J}^{ij}_{\alpha\beta}$):

$$
\begin{aligned}
J^{ij}_{\alpha\beta} &= J \sum_k \epsilon_{ijk} \sum_\gamma \epsilon_{\alpha\beta\gamma} \; , \\
\tilde{J}^{ij}_{\alpha\beta} &= \tilde{J} \sum_k \epsilon_{ijk} \sum_\gamma \epsilon_{\alpha\beta\gamma} \; ,
\end{aligned}
\tag{15}
$$

where (i, j, k) and (α, β, γ) run over $(1, 2, 3)$ and (e, μ, τ), respectively. It is a unique feature of the three-family flavor mixing scenario, for either leptons or quarks [10], that there exists a universal CP-violating parameter.

To establish the relationship between $\tilde{J}^{ij}_{\alpha\beta}$ and $J^{ij}_{\alpha\beta}$, we make use of th equality in Eq. (11). The key point is that the imaginary parts of the rephasing-invariant quantities $Z_{\alpha\beta}Z_{\beta\gamma}Z_{\gamma\alpha}$ and $\tilde{Z}_{\alpha\beta}\tilde{Z}_{\beta\gamma}\tilde{Z}_{\gamma\alpha}$,

$$
\begin{aligned}
{\rm Im}(Z_{\alpha\beta}Z_{\beta\gamma}Z_{\gamma\alpha}) &= \sum^3_{i=1}\sum^3_{j=1}\sum^3_{k=1} \left[\Delta_{i0}\Delta_{j0}\Delta_{k0} \; {\rm Im}\left(V_{\alpha i}V_{\beta j}V_{\gamma k}V^*_{\alpha k}V^*_{\beta i}V^*_{\gamma j}\right)\right] \; , \\
{\rm Im}(\tilde{Z}_{\alpha\beta}\tilde{Z}_{\beta\gamma}\tilde{Z}_{\gamma\alpha}) &= \sum^3_{i=1}\sum^3_{j=1}\sum^3_{k=1} \left[\tilde{\Delta}_{i0}\tilde{\Delta}_{j0}\tilde{\Delta}_{k0} \; {\rm Im}\left(\tilde{V}_{\alpha i}\tilde{V}_{\beta j}\tilde{V}_{\gamma k}\tilde{V}^*_{\alpha k}\tilde{V}^*_{\beta i}\tilde{V}^*_{\gamma j}\right)\right] \; ,
\end{aligned}
\tag{16}
$$

which do not vanish unless leptonic CP and T are good symmetries, amount to each other. The right-hand side of Eq. (16) can be expanded in terms of $J^{ij}_{\alpha\beta}$ and $\tilde{J}^{ij}_{\alpha\beta}$. In doing so, one needs to use Eqs. (12), (13) and (14) as well as the unitarity conditions of V and $\tilde{V}$ frequently. After some lengthy but straightforward algebraic calculations, we arrive at the following sum rules of CP- or T-violating parameters:

$$
\begin{aligned}
&\tilde{\Delta}_{10}\tilde{\Delta}_{20}\tilde{\Delta}_{30}\sum_{i=1}^{3}\left(\tilde{J}^{0i}_{\alpha\beta}|\tilde{V}_{\gamma i}|^2+\tilde{J}^{0i}_{\beta\gamma}|\tilde{V}_{\alpha i}|^2+\tilde{J}^{0i}_{\gamma\alpha}|\tilde{V}_{\beta i}|^2\right)\\
&+\sum_{i=1}^{3}\sum_{j=1}^{3}\left[\tilde{\Delta}_{i0}\tilde{\Delta}^2_{j0}\left(\tilde{J}^{ij}_{\alpha\beta}|\tilde{V}_{\gamma j}|^2+\tilde{J}^{ij}_{\beta\gamma}|\tilde{V}_{\alpha j}|^2+\tilde{J}^{ij}_{\gamma\alpha}|\tilde{V}_{\beta j}|^2\right)\right]\\
=\;&\Delta_{10}\Delta_{20}\Delta_{30}\sum_{i=1}^{3}\left(J^{0i}_{\alpha\beta}|V_{\gamma i}|^2+J^{0i}_{\beta\gamma}|V_{\alpha i}|^2+J^{0i}_{\gamma\alpha}|V_{\beta i}|^2\right)\\
&+\sum_{i=1}^{3}\sum_{j=1}^{3}\left[\Delta_{i0}\Delta^2_{j0}\left(J^{ij}_{\alpha\beta}|V_{\gamma j}|^2+J^{ij}_{\beta\gamma}|V_{\alpha j}|^2+J^{ij}_{\gamma\alpha}|V_{\beta j}|^2\right)\right]\ . \qquad (17)
\end{aligned}
$$

We remark that this new result is model-independent and rephasing-invariant. It may be considerably simplified, once the hierarchy of neutrino masses and that of flavor mixing angles are theoretically assumed or experimentally measured. If one "switches off" the mass of the sterile neutrino and its mixing with active neutrinos (i.e., $a' = 0$, $\Delta_{i0} = m_i^2$, $\tilde{\Delta}_{i0} = \tilde{m}_i^2$, $J^{0i}_{\alpha\beta} = 0$, and $\tilde{J}^{0i}_{\alpha\beta} = 0$), then Eq. (17) turns out to take the form

$$
\tilde{J}\tilde{\Delta}_{21}\tilde{\Delta}_{31}\tilde{\Delta}_{32} \;=\; J\Delta_{21}\Delta_{31}\Delta_{32}\ . \qquad (18)
$$

This elegant relationship has been derived in Refs. [7,11] with the help of the equality $\mathrm{Det}(\tilde{C}) = \mathrm{Det}(C)$, instead of Eq. (11), in the three-neutrino mixing scheme.

The matter-corrected CP-violating parameters $\tilde{J}^{ij}_{\alpha\beta}$ can, at least in principle, be determined from the measurement of CP- and T-violating effects in a variety of long-baseline neutrino oscillation experiments. The conversion probability of a neutrino ν_α to another neutrino ν_β is given in matter as

$$
\tilde{P}(\nu_\alpha\rightarrow\nu_\beta) \;=\; \delta_{\alpha\beta} - 4\sum_{i<j}\left[\mathrm{Re}\left(\tilde{V}_{\alpha i}\tilde{V}_{\beta j}\tilde{V}^*_{\alpha j}\tilde{V}^*_{\beta i}\right)\sin^2\tilde{F}_{ji}\right] - 2\sum_{i<j}\left[\tilde{J}^{ij}_{\alpha\beta}\sin(2\tilde{F}_{ji})\right]\ , \qquad (19)
$$

where $\tilde{F}_{ji} \equiv 1.27\tilde{\Delta}_{ji}L/E$ with $\tilde{\Delta}_{ji} \equiv \tilde{m}_j^2 - \tilde{m}_i^2$, L stands for the baseline length (in unit of km), and E is the neutrino beam energy (in unit of GeV). The transition probability $\tilde{P}(\nu_\beta\rightarrow\nu_\alpha)$ can directly be read off from Eq. (19), if the replacements $\tilde{J}^{ij}_{\alpha\beta} \Longrightarrow -\tilde{J}^{ij}_{\alpha\beta}$ are made [1]. To obtain the probability $\tilde{P}(\overline{\nu}_\alpha\rightarrow\overline{\nu}_\beta)$, however, both the replacements $J^{ij}_{\alpha\beta} \Longrightarrow -J^{ij}_{\alpha\beta}$ and $(a, a') \Longrightarrow (-a, -a')$ need be made for Eq. (19). In this case, $\tilde{P}(\overline{\nu}_\alpha\rightarrow\overline{\nu}_\beta)$ is not equal to $\tilde{P}(\nu_\beta\rightarrow\nu_\alpha)$. The difference between $\tilde{P}(\overline{\nu}_\alpha\rightarrow\overline{\nu}_\beta)$ and $\tilde{P}(\nu_\beta\rightarrow\nu_\alpha)$ is a false signal of CPT violation, induced actually by the matter effect [5]. Thus the CP-violating asymmetries between $\tilde{P}(\nu_\alpha\rightarrow\nu_\beta)$ and $\tilde{P}(\overline{\nu}_\alpha\rightarrow\overline{\nu}_\beta)$ are in general different from the T-violating asymmetries between $\tilde{P}(\nu_\alpha\rightarrow\nu_\beta)$ and $\tilde{P}(\nu_\beta\rightarrow\nu_\alpha)$. The latter can be explicitly expressed as follows:

---

[1]Note that the differences of effective neutrino masses $\tilde{\Delta}_{i0}$ (for $i = 1, 2, 3$), which must be CP-conserving, keep unchanged for the replacements $J^{ij}_{\alpha\beta} \Longrightarrow -J^{ij}_{\alpha\beta}$. Therefore the sign flip of $J^{ij}_{\alpha\beta}$ results in that of $\tilde{J}^{ij}_{\alpha\beta}$, as indicated by the sum rules given in Eq. (17).

$$
\begin{aligned}
\Delta\tilde{P}_{\alpha\beta} & \equiv \tilde{P}(\nu_\beta \rightarrow \nu_\alpha) - \tilde{P}(\nu_\alpha \rightarrow \nu_\beta) \\
& = 4\Big[\tilde{J}^{01}_{\alpha\beta}\sin(2\tilde{F}_{10}) + \tilde{J}^{02}_{\alpha\beta}\sin(2\tilde{F}_{20}) + \tilde{J}^{03}_{\alpha\beta}\sin(2\tilde{F}_{30}) \\
& \quad + \tilde{J}^{12}_{\alpha\beta}\sin(2\tilde{F}_{21}) + \tilde{J}^{13}_{\alpha\beta}\sin(2\tilde{F}_{31}) + \tilde{J}^{23}_{\alpha\beta}\sin(2\tilde{F}_{32})\Big] \; .
\end{aligned}
\tag{20}
$$

If the hierarchical patterns of neutrino masses and flavor mixing angles are assumed, the expression of $\Delta\tilde{P}_{\alpha\beta}$ may somehow be simplified [12]. Note that only three of the twelve nonvanishing asymmetries $\Delta\tilde{P}_{\alpha\beta}$ are independent, as a consequence of the unitarity of $\tilde{V}$ or the correlation of $\tilde{J}^{ij}_{\alpha\beta}$. Since only the transition probabilities of active neutrinos can be realistically measured, we are more interested in the T-violating asymmetries $\Delta\tilde{P}_{e\mu}$, $\Delta\tilde{P}_{\mu\tau}$ and $\Delta\tilde{P}_{\tau e}$. These three measurables, which are independent of one another in the four-neutrino mixing scheme under discussion, must be identical in the conventional three-neutrino mixing scheme. In the latter case, where $a' = 0$, $\tilde{J}^{01}_{\alpha\beta} = \tilde{J}^{02}_{\alpha\beta} = \tilde{J}^{03}_{\alpha\beta} = 0$, and $\tilde{J}^{12}_{\alpha\beta} = -\tilde{J}^{13}_{\alpha\beta} = \tilde{J}^{23}_{\alpha\beta} = \tilde{J}$ for (α, β) running over (e, μ), (μ, τ) and (τ, e), Eq. (20) can be simplified to

$$
\Delta\tilde{P}^{(3)}_{\alpha\beta} = 16\tilde{J}\sin\tilde{F}_{21}\sin\tilde{F}_{31}\sin\tilde{F}_{32} \; . \tag{21}
$$

The overall matter contamination residing in $\Delta\tilde{P}_{\alpha\beta}$ is usually expected to be insignificant. The reason is simply that the terrestrial matter effects in $\tilde{P}(\nu_\alpha \rightarrow \nu_\beta)$ and $\tilde{P}(\nu_\beta \rightarrow \nu_\alpha)$, which both depend on the parameters (a, a'), may partly (even essentially) cancel each other in the T-violating asymmetry $\Delta\tilde{P}_{\alpha\beta}$. In contrast, $\tilde{P}(\nu_\alpha \rightarrow \nu_\beta)$ and $\tilde{P}(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta)$ are associated respectively with $(+a, +a')$ and $(-a, -a')$, thus there should not have large cancellation of matter effects in the corresponding CP-violating asymmetries.

4 In summary, we have introduced the commutators of lepton mass matrices to describe the flavor mixing phenomenon of three active and one sterile neutrinos. It has been shown that the commutator defined in vacuum is invariant under terrestrial matter effects. An important consequence of this interesting result is the emergence of a set of model-independent sum rules for neutrino masses in two different media. Such sum rules may even be generalized to hold for an arbitrary number of neutrino families. We have also uncovered some useful sum rules for the rephasing-invariant measures of leptonic CP violation in the four-neutrino mixing scheme. A generic formula of T-violating asymmetries, which is applicable in particular to the future long-baseline neutrino oscillation experiments, has been derived and discussed.

The remarkable strategy of this paper is to formulate the sum rules of neutrino masses and CP violation in a model-independent way. Hence we have concentrated upon the generic formalism instead of the specific scenarios. The relevant results are anticipated to be a useful addition to the phenomenology of lepton flavor mixing and neutrino oscillations, and the application of them to the realistic long-baseline neutrino experiments deserves a further study elsewhere.

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